# Exploring the Science Framework

## *Making connections in math with the Common Core State Standards*

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The vision for science education set forth in *A Framework for K–12 Science Education* (NRC 2012) makes it clear that for today's students to become the scientifically literate citizens of tomorrow their educational experiences must help them become mathematically proficient. "The focus here is on important practices, such as modeling, developing explanations, and engaging in critique and evaluation" (NRC 2012, p. 3–2). Mathematics is fundamental to modeling and providing evidence-based conclusions. The *Framework* also includes "using mathematics, information and computer technology, and computational thinking" in its list of eight essential practices for K–12 science and mathematics (NRC 2012, p 3–5). But what does it mean for students to become mathematically proficient in the context of science? And how can science teachers help students develop that proficiency? This article addresses these questions.

To many Americans, mathematical proficiency means being able to robotically calculate or apply algorithms. Yet, the Common Core State Standards (CCSS) highlight a very different view. "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (National Governors Association Center for Best Practices 2010, p. 7). It's this view of mathematical proficiency that permeates the three dimensions of science presented in the *Framework*.

Inherent in this view is quantitative reasoning, which includes (a) the act of quantification where students identify variables within a context, with attributed units of measure, (b) the use of mathematical concepts in ways that enable description, manipulation, and the generation of claims from quantifiable variables, (c) the use of mathematical models to discover trends and make predictions, and (d) the creation and revision of mathematical representations of phenomena (Mayes, Peterson, and Bonilla 2012).

### Science, engineering, and mathematical practices

To provide a glimpse of how this view of mathematical proficiency will become an important element of the future science education of K–12 students, let's focus on the *Framework's* Scientific and Engineering Practices and the Mathematical Practices of the Common Core State Standards (CCSS-M). (Their alignment is shown in Figure 1.)

## Asking and investigating questions

Developing students' ability to ask well-formulated questions is basic to both science and engineering (Practice 1) and mathematics (Practice 1). The CCSS-M call for students to be able to determine the meaning of a problem and find entry points to its solution, which requires analyzing givens, constraints, relationships, and goals, with the purpose of making conjectures (i.e., formulating hypotheses) to be tested. Just as science requires formulation and refinement of questions so they can be answered empirically, mathematics attends to questions that may be quantified and then addressed mathematically. Making sense of problems and persevering in solving them (Mathematical Practices [MP] 1) calls for the conjectures to be followed by planning a means to reach a solution. Students should consider analogous problems, test special cases, and decompose the problem into simpler cases. This parallels the science focus on designing experimental or observational inquiries (planning and carrying out investigations, Science and Engineering Practices [SEP] 3). This process begins by quantifying the situation being studied through identifying variables and considering how they can be observed, measured, and controlled, as well as considering confounding variables. Students should be engaged in investigations that emerge from their own questions about real-world grand challenges, such as availability and uses of energy resources or biodiversity loss, which are related to their community or region. The interdisciplinary nature of such questions will lead naturally to linkages between science and mathematics.

## Problem solving

Making sense of problems and persevering in solving them (MP 2) calls for students to make sense of quantities and

**FIGURE 1**

**Alignment between mathematical practices and scientific and engineering practices.**

| Mathematical Practices (MP) | Science and Engineering Practices (SEP) |
|---|---|
| 1. Making sense of problems and persevering in solving them | 1. Asking questions and defining problems<br>3. Planning and carrying out investigations |
| 2. Reason abstractly and quantitatively | 2. Developing and using models<br>3. Planning and carrying out investigations<br>5. Using mathematics and computational thinking |
| 3. Construct viable arguments and critique the reasoning of others | 5. Using mathematics and computational thinking<br>6. Constructing explanations and designing solutions<br>7. Engaging in argument from evidence<br>8. Obtaining, evaluating, and communicating information |
| 4. Model with mathematics | 2. Developing and using models<br>3. Planning and carrying out investigations |
| 5. Use appropriate tools strategically | 2. Developing and using models<br>3. Planning and carrying out investigations<br>4. Analyzing and interpreting data |
| 6. Attend to precision | 3. Planning and carrying out investigations<br>8. Obtaining, evaluating, and communicating information |
| 7. Look for and make use of structure | 4. Analyzing and interpreting data<br>6. Constructing explanations and designing solutions<br>7. Engaging in argument from evidence |
| 8. Looking for and expressing regularity in repeated reasoning | 5. Using mathematics and computational thinking<br>6. Constructing explanations and designing solutions |

their relationships in problem situations. This typically unfolds in three steps:

1. Students must be able to identify quantities within a scientific context, then represent the situation symbolically. This sets the stage for manipulating the variable quantities using rules of mathematics.
2. Students must continually relate the variables and mathematical representations to the science context so the manipulations they perform move them closer to answering the posed question.
3. Students must move back to the scientific context to provide a data-based solution to the problem.

The act of quantification is essential to creating variables and ensuring that a variable has attributes and measure. It deserves more attention in science classrooms than currently given.

## Models and modeling

Both the *Framework* and CCSS-M call for a focus on modeling. Models as discussed in the *Framework* are more broadly construed as diagrams, physical replicas, analogies, computer simulations, and mathematical representations. CCSS-M emphasizes abstract mathematical reasoning and quantitative reasoning with the goal of developing an abstract mathematical model such as an equation or function.

Not all models are necessarily quantitative, and quantitative models can take on many different representations beyond that of an equation. Quantitative models can be tables of data, graphs of relationships, statistical displays such as pie graphs, and pictorial science models such as the carbon cycle model shown in Figure 2, adapted from one by the GLOBE Carbon Cycle Project. In addition, science often has embedded variables in models, something outside the experience of students in a mathematics class where typically only two variables are displayed.

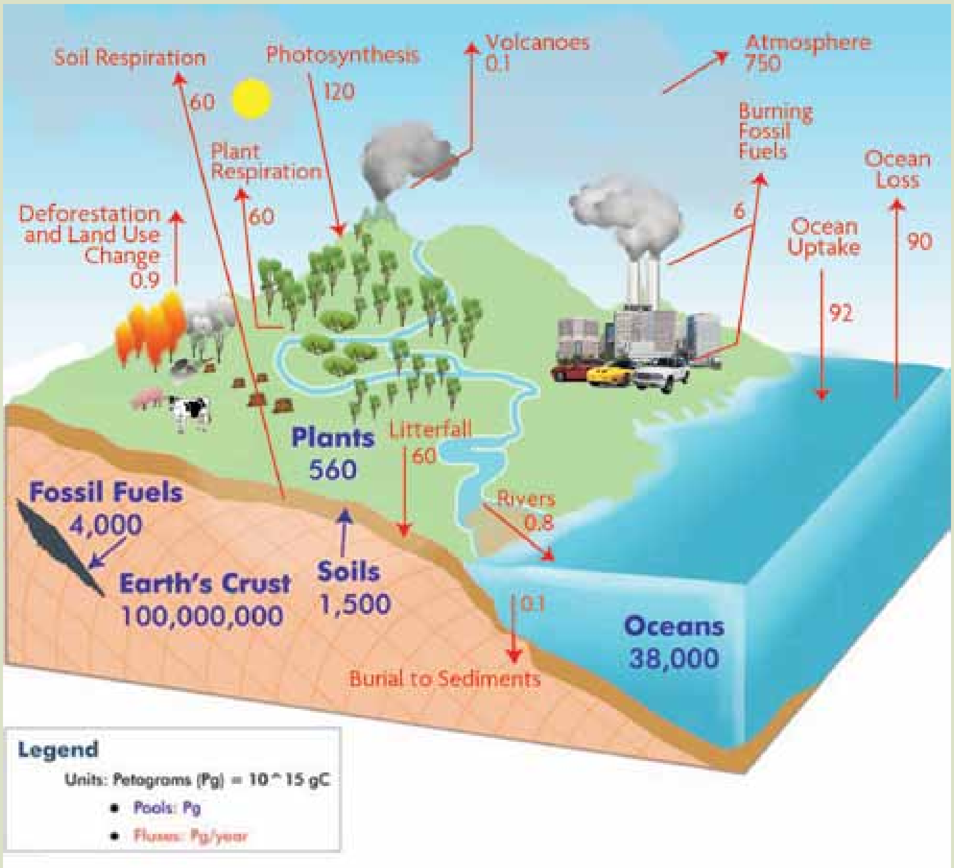
Finally, in mathematics, students are too often provided with data or with the equation modeling data without engaging them in collecting data. Data collection is a scientific and engineering practice (planning and carrying out investigations, SEP 3) that is a natural extension of the investigative design process. The design process involves students determining what data are to be gathered, what instruments are needed to measure data, how much data are needed to address reliability and precision concerns, and what experimental procedures to follow. The processes of designing investigations and collecting data have the potential to engage students in all the mathematical practices, 1 through 6.

## Argumentation

Constructing viable arguments and critiquing the reasoning of others (MP 3) is analogous to the scientific and engineering practice of engaging in argument from evidence (SEP 7). Both emphasize justifying claims in an argument by grounding them in evidence (e.g., mathematical or scientific theories) that is accepted by the scientific or mathematical community. Students should critique their own arguments, identifying weaknesses and flaws in logic, revise their arguments, and submit them to peers

**FIGURE 2**

**Carbon cycle model.**



for review. The ultimate form of argument in mathematics is the abstract proof, but truly understanding a proof requires experimentation, just as in science. Data analysis is fundamental to argumentation in science and engineering, but it often requires quantitative analysis of data within a context. Both science and mathematics value the ability to compare the effectiveness of two plausible arguments and distinguish correct logical reasoning from flawed reasoning.

In addition to constructing an argument, communicating it to others is paramount (SEP 8). The mathematical practices of constructing viable arguments and critiquing the reasoning of others (MP 3) and attending to precision (MP 6) have elements of communication, the ability to communicate precisely and clearly a mathematical argument to peers and experts. MP 6 highlights the special difficulties in communicating mathematics due to the intensively symbolic nature of the subject as well as the density of language in mathematical texts. SEP 8 focuses on related communication issues, specifically attending to the difficulty students have with reading scientific and engineering texts and primary literature, as well as with scientific writing. The difficulty of communicating in science and mathematics is compounded by the fact that scientific ideas are often represented quantitatively as tables, graphs, charts, equations, and symbols.

## Mathematical tools use

The selection of appropriate mathematical tools for certain tasks is the focus of MP 5. In relation to science, the toolbox includes selection of the appropriate type of model (SEP 2). Students must understand the limitations and precision of the selected model type, for example, using a graphic representation rather than an equation. It also means the ability to select the appropriate mathematical algorithm as a tool to analyze data within a science context. It includes the type of instrument used, such as selecting a protractor, calculator with remote light sensor, spreadsheet, computer algebra system, statistical package, dynamic geometry software, or computer simulation. In addition, science and engineering have an extensive set of instruments to select from to measure quantities (planning and carrying out investigations, SEP 3), which raises concerns of precision, accuracy, and error. Once data are collected, they need to be analyzed and interpreted (SEP 4). Mathematics is essential for expressing relationships in the data. Students of science tell the story of data using descriptive statistics, test hypotheses using statistical analysis, and explore causal and correlational relationships.

The toolbox for science and engineering is ever expanding, with the advent of two new paradigms. The paradigms of empirical methods (applied or experimental science) and theory (theoretical science) were until recently considered the two legs of science. But over the last 20 years, due to increasing computing capabilities, two new paradigms have arisen: computational science (scientific computing) and data-intensive science (data-centric science) (Hey, Tansley, and Tolle 2009).

Computational science is embedded in mathematics, science and engineering, and the humanities; it complements the empirical methods and theory paradigms but does not replace them. The goal of scientific computing is to improve the understanding of physical phenomena. Scientific computing focuses on simulations and modeling to provide both qualitative and quantitative insights into complex systems and phenomena that would be too expensive, dangerous, or even impossible to study by direct experimentation or theoretical methods (Turner et al. 2011). The explosion of data in the 21st century led to the invention of data-intensive science as a fourth paradigm, which focuses on compressed sensing (effective use of large data sets), curation (data storage issues), analysis and modeling (mining the data), and visualization (effective human-computer interface). SEP 5 highlights science and engineering education issues related to these two new paradigms.

## Mathematical structure

The ability to look for and make use of structure (MP 7) and look for and express regularity in repeated reasoning (MP 8) focus on abstract mathematical argumentation. For example, a student who can see the structure of the distributive property $a(b+c) = ab + ac$ in the expression $(x+y)(b+c) = (x+y)b + (x+y)c$ does not need to memorize rules for multiplying binomial expressions. Use of structure and repeated reasoning most closely align with constructing explanations and designing solutions (SEP 6). We consider these to be the more theoretical aspects of the mathematics and science processes, while the other processes are more experimental in nature. Students need to engage with standard scientific explanations of the world that link science theory with specific observations.

While we find a lot of commonality between the practices put forth in the *Framework* and the CCSS-M, we share concerns similar to those discussed in a review of the Summer 2012 draft of the Next Generation Science Standards (NGSS) conducted by the Fordham Institute (Gross et al. 2012). In brief, the Fordham Institute reviewers revealed the need for mathematics content specificity in the NGSS and for vigilance in the alignment of the NGSS and the CCSS-M to achieve the desired dovetailing of science and mathematics learning across the grade levels.

We believe it is only through careful attention to the specific science, engineering, and mathematics concepts to be learned and the alignment of them across the grade

FIGURE 3

## Examples of *Framework* and CCSS-M alignment for global climate change.

| Grade Level | Framework for K–12 Science | Common Core State Standards–Mathematics |
|---|---|---|
| **Grade 2** | By the end of grade 2 students should know: "Weather is the combination of sunlight, wind, snow or rain, and temperature in a particular region at a particular time. People measure these conditions to describe and record weather and to notice patterns over time" (NRC 2012, p. 188). | The CCSS-M have 2nd graders solving problems involving addition and subtraction within 100, understanding place value up to 1,000, recognizing the need for standard units of measure of length, representing and interpreting data, and reasoning with basic shapes and their attributes. |
| **Grade 5** | By the end of 5th grade the expectation for global climate change is: "If Earth's global mean temperature continues to rise, the lives of humans and organisms will be affected in many different ways" (NRC 2012, p. 98). | The CCSS-M has fifth graders writing and interpreting numerical expressions, analyzing patterns and relationships, performing operations with multi-digit whole numbers and decimals to hundredths, using equivalent fractions to add and subtract fractions, multiplying and dividing fractions, converting measurement units within a given measurement system, measuring volume, representing and interpreting data, graphing points on the coordinate plane to solve real-world problems, and classifying two-dimensional figures into categories based on their properties. |
| **Grade 8** | The end of 8th grade expectation for climate change is to understand that human activities, such as carbon dioxide release from burning fuels, are major factors in global warming. Reducing the level of climate change requires an understanding of climate science, engineering capabilities, and human behavior (NRC 2012, p. 198). | The CCSS-M 8th grade standards include awareness of numbers beyond the rational numbers, work with radicals and integer exponents, proportional relationships, ability to analyze and solve linear equations and systems of linear equations, use linear functions to model relationships between quantities, understand congruence and similarity, the Pythagorean Theorem, solve real-world problems involving volume of cylinders, cones, and spheres, and use statistics to investigate patterns of association in bivariate data. |
| **Grade 12** | By the end of high school students should understand that climate change is slow and difficult to recognize without studying long-term trends, such as studying past climate patterns. Computer simulations are providing a new lens for researching climate change, revealing important discoveries about how the ocean, the atmosphere, and the biosphere interact and are modified in response to human activity (NRC 2012, p. 198). | The CCSS-M high school standards are by conceptual categories not grade level. The conceptual categories of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability specify the mathematics that all students should study in order to be college and career ready. Functions are expanded to include quadratic, exponential, and trigonometric functions, broadening the potential models for science. |

levels that the vision for science and engineering teaching and learning presented in the *Framework* can be realized. In our examples that follow, we attempt to illustrate how this attention and alignment might be enacted.

## Examples of *Framework* and CCSS-M alignment

The core Earth and Space Science idea of Earth and Human Activity provides a good context for showcasing grand challenges that students can explore in their own community. Concepts central to this core idea include natural resources, natural hazards, human impact on Earth systems, and global climate change. Change is a core quantitative concept, so we chose global climate change as our focus concept. Following the lead of the *Framework*, we discuss science tasks that could be accomplished by the end of grades 2, 5, 8, and 12. The endpoints for these grades described in the CCSS-M and in the *Framework* for Global Climate Change are presented in Figure 3 (p. 31).

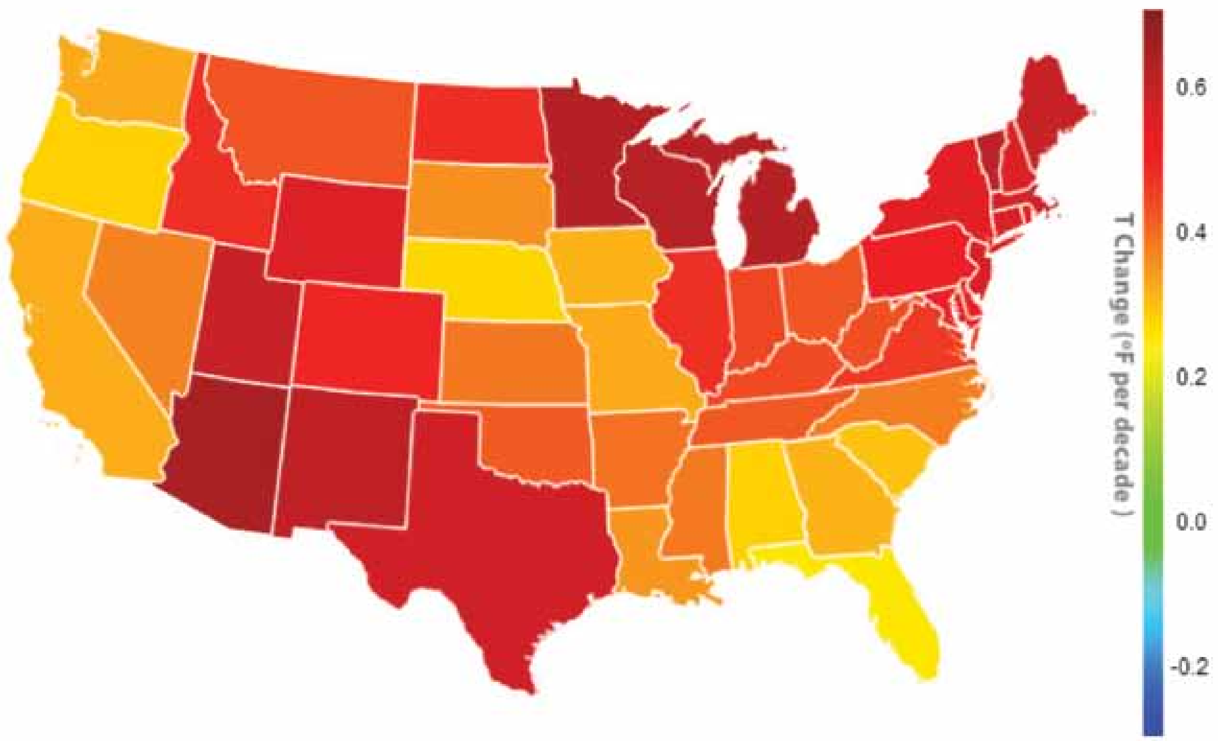
### *Grade 2*

Climate is not a grade 2 concept. However, prerequisite understandings are developed through the study of weather. Weather tasks for second graders that address both sets of understandings might involve students observing television weather reports followed by drawing pictures of and describing things they believe make up the weather. These experiences will enable students to construct their own definitions of weather and list variables that make up weather, such as rain, sunshine, and wind.

Subsequent learning experiences might involve students collecting and measuring rain to the nearest centimeter for each month of the school year for their community or being given these data. Then, students could be asked to draw pictures representing rain by month; this may be a bar graph or a dot chart using M&M candies. Using visual data displays, student could answer questions about specific weather variables: Which month was the wettest? The driest? Conclude by having students link their findings to the

**FIGURE 4**

**National temperature trends (Grade 5).**



Average annual temperatures have been rising in every state since 1970. This map shows how fast each state has been warming each decade over the past 42 years.

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context of the local environment through such questions as these: What do you think happened to plants in the months with low rainfall? What other weather conditions interact with the amount of rain to affect plant life?

### *Grade 5*

Climate Change tasks for fifth grade may have students considering data on state, national, and international annual temperature changes. For example, students could be asked to examine Climate Central's national map on temperature change (Figure 4; also see "On the web"). Questions to prompt their interactions with the map could include: What percentage of states has warmed more than 0.2 degrees each decade over the past 40 years? How much has the state you lived in warmed?

Further investigations might have students examining data for the state in which they live. For example, students could be directed to one of the red points on the graph representing Georgia (Figure 5) and asked to interpret what it means. What does the general trend of the scatter plot of points indicate? Further using the information presented in the figure, students might be asked to measure the temperature each day for a week to the nearest 0.1 degree. What can you say about natural flux in daily temperatures and how it relates to the annual average temperature? If the temperature continues to increase at the current rate, what will the average temperature be in 20 years? What potential impact does this warming trend have in your state? Sample responses could be decreased biodiversity due to extinctions, agricultural economic impact, and increased heat-related problems for the football team.

### *Grade 8*

Climate Change tasks for eighth-grade students could be initiated by extending the discussion of the Georgia warming data. Provide students with the data for average annual temperature per year for the state in a table, then have them plot the data and construct a scatter plot like the one in Figure 5. The plot could then be used to address questions such as these: What is the trend of the data in this scatter plot? Is it decreasing or increasing? Estimate a line of best fit for the data that represents the trend. Discussing with students what is needed to determine a line, slope, and a point may help them accomplish this. A potential point for the line is the center point of the data set, which students can calculate as the ordered pair with $x$-intercept (the average of the first coordinate values of the data points in the set) and the $y$-intercept (the average of the second coordinate values).

Once students have determined the center point of the data set, ask them to place a ruler on the center point and vary the slope by rotating the ruler about the center point to best represent the trend of the data. Then, have them write out the equation of the line and use the linear model to predict temperatures for future years. Conclude by helping students relate this back to the science context, using such questions as: What variables can we control to reduce or stabilize the temperature trend? Among the possible variables is carbon dioxide, which, if controlled or reduced, would reduce greenhouse gases in the atmosphere and may impact climate change.

### *Grade 12*

Climate Change tasks for grade 12 students may involve revisiting the scatter plot of state temperature data. But, this time ask students to provide a power function model or exponential model for the data. Rich discussions of which function is the best model for the data would engage students in exploring error and best-fit concepts.

Carbon dioxide as a mitigating factor in global climate change can be explored in more depth. For example, Figure 6 (p. 34) provides data on historic trends in atmospheric carbon dioxide. Ask students to quantitatively interpret the trends in the graph as naturally occurring cycles. The claim has been made that today the Earth is experiencing just a phase in a natural cycle of carbon dioxide change. Students could be challenged to interpret the data for evidence that

**FIGURE 5**

**Georgia temperature trends (Grade 8).**

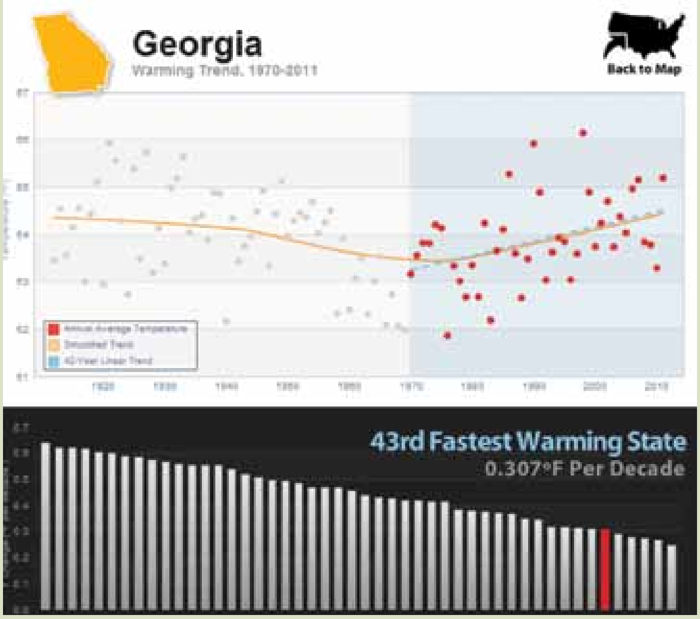

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**FIGURE 6**

**Atmospheric carbon dioxide flux (Grade 12).**

supports this claim. Questions that could serve to guide students' work include: How were the data collected? Are the data reliable? What are likely causes of the fluxes in atmospheric carbon dioxide?

## Looking forward

There is much work yet be done to specify the science, engineering, and mathematics concepts to be learned by K–12 students and align them across the grade levels, but there is great promise in this work. The approaching release of the NGSS on the heels of the CCSS-M facilitates this work, making alignment and compatible pacing of expected learning outcomes across the grade levels possible. Both the *Framework* and the CCSS-M emphasize student construction of conceptual understandings and the development of real-world practices. Indeed, mathematical proficiency in the context of science highlights the application of mathematics to solve real-world problems in everyday life, society, and the workplace.

Through the *Framework* we see glimpses of curricula that are problem-based and community-focused to increase student engagement and that take interdisciplinary approaches to the grand challenges of today and tomorrow. These curricula embrace mathematical proficiency as a desirable outcome of science and engineering education. Students must build mathematical proficiency as they develop understandings and skills in science and engineering that will enable them to become the scientifically literate citizens of tomorrow.

Helping students develop the mathematical proficiency described in the *Framework* will be challenging. However, you can take steps to enhance the odds of your success. We recommend that science teachers read the CCSS-M and upon their release, the NGSS, focusing on the guidance most pertinent to the grade level or high school courses you teach. Look for points of alignment and compatibility of pacing and determine what will work for your school context and students. The information presented on grade endpoints in CCSS-M and the *Framework* may serve as an advance organizer for this exploration of the standards documents. We also encourage meeting regularly with mathematics teachers to discuss expectations for student learning and work collaboratively to build lessons and units.

As you progress with this work, recognize that while standards offer guidance, it is teachers, through planning and instruction, who enact the vision for student success set forth in the standards. Finally, we recommend that science teachers strengthen their own understandings of the mathematics germane to the science they teach. This can be done by establishing professional learning communities (PLC) that are interdisciplinary, including both science and mathematics teachers. The PLC can review the NGSS and CCSS-M standards together, selecting real-world grand challenges to engage students in cross-discipline, problem-based episodes. School administrations can support the process by providing common planning time and, when possible, team teaching of STEM courses. In addition, the PLC can reach out to regional higher education institutions and STEM research centers to seek mentoring from scientists and mathematicians on STEM content. ■

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## On the web

Climate Central national map: *www.climatecentral.org/news/the-heat-is-on*

## References

Gross, P.R., L.S. Lerner, J. Lynch, M. Schwartz, R. Schwartz, and W.S. Wilson. 2012. *Commentary & Feedback on Draft I of the Next Generation Science Standards.* Fordham Institute.

Hey, T., S. Tansley, and K. Tolle, eds. 2009. *The Fourth Paradigm: Data Intensive Scientific Discovery*. Redmond, WA: Microsoft Research.

Mayes, R., F. Peterson, and R. Bonilla. 2012. Quantitative reasoning: Current state of understanding. In *WISDOM$^e$: Quantitative reasoning and mathematical modeling: A driver for STEM integrated education and teaching in context,* eds. R. Mayes and L. Hatfield, 7–38. Laramie, WY: University of Wyoming.

National Governors Association Center for Best Practices, Council of Chief State School Officers. 2010. *Common Core State Standards (Mathematics)*. National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington, DC.

National Research Council (NRC). 2012. *A framework for K–12 science education: Practices, crosscutting concepts, and core ideas*. Washington, DC: National Academies Press.

Turner, P., et al. 2011. Undergraduate computational science and engineering education. *Society for Industrial and Applied Mathematics Review* (SIAM Rev.) 53 (3): 561–574.